continuous slowing down model. The spectrum of the positrons escaping the surface of a plane source thicker than the positron range is not identical with the slowing-down spectrum in an infinite medium due to a deficiency in the number of positrons with low energies.

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Relative Energy Loss to Optical and Acoustic Modes of Electrons in Avalanche Breakdown in Ge

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Expressions are derived for the rates of loss of high-energy electrons in a many-valley band structure to acoustic and optical modes. Evaluating these for germanium, for which all the parameters are now reasonably well known, we find that, contrary to what is usually assumed, the rate of loss to acoustic modes is greater than that to optical modes for electron energies of the order of the energy gap.

IT has been customary to assume, rather arbitrarily, in calculations on avalanche breakdown in germanium and silicon that, just below the threshold for ionization, the predominant energy loss process for electrons and holes is optical phonon emission.[1–3] It has been realized for some time that for both electrons and holes in germanium, contrary to earlier expectations, the rate of loss to optical phonons exceeds that to acoustic phonons over a wide range of fields starting from low ones.[4,5] However, in the limit of high enough fields, or high enough carrier energies, the loss to acoustic modes must once again predominate.[5,6] It is the purpose of the present note to demonstrate that in Ge, for electrons with energies of the order of the band gap and larger, the rate of loss to acoustic modes is larger than that to optical modes.

Using the usual perturbation theory, we may write for the rate of energy loss to acoustic modes of an electron with wave vector $\mathbf{k}$, measured relative to the band edge,

$$\frac{d\mathcal{E}}{dt}(\mathbf{k})=\frac{2\pi}{\hbar}\sum_{\mathbf{q}}\sum_{\alpha}[\hbar\omega_{q\alpha}|(\mathbf{k}+\mathbf{q},N_{\mathbf{q}\alpha}-1|H_\alpha'|\mathbf{k},N_{\mathbf{q}\alpha})|^2 \times\delta(\mathcal{E}_{\mathbf{k}+\mathbf{q}}-\hbar\omega_{q\alpha}-\mathcal{E}_{\mathbf{k}})-\hbar\omega_{q\alpha}|(\mathbf{k}-\mathbf{q},N_{\mathbf{q}\alpha}+1 \times|H_\alpha'|\mathbf{k},N_{\mathbf{q}\alpha})|^2\delta(\mathcal{E}_{\mathbf{k}-\mathbf{q}}+\hbar\omega_{q\alpha}-\mathcal{E}_{\mathbf{k}})], \quad (1)$$

where $\hbar\omega_{q\alpha}$ and $N_{\mathbf{q}\alpha}$ are the energy and steady-state number of phonons with wave vector $\mathbf{q}$ and polarization α. The argument of the δ function is the difference between the initial and final energies of the system. The first term of (1) gives the rate of energy gain due to phonon absorption, while the second term gives the rate of loss due to emission. The matrix elements for acoustic phonon interaction in the many-valley band structure are given by[7]

$$|(\mathbf{k}\pm\mathbf{q}|H_\alpha'|\mathbf{k})|^2=\frac{\Xi_\alpha^2\hbar q}{2V\rho u_\alpha}\left(N_{q\alpha}+\tfrac{1}{2}+\frac{\delta N_{q\alpha}}{2}\right), \quad (2)$$

where for longitudinal waves $\alpha=l$, $\Xi_l=\Xi_d+\Xi_u\cos^2\theta$, and for transverse waves $\alpha=t$, $\Xi_t=\Xi_u\sin\theta\cos\theta$, θ being the angle between $\mathbf{q}$ and the z (longitudinal) axis of the constant-energy ellipsoids. The quantity u_α is an average velocity for acoustic waves of polarization α, and V and ρ are the volume and density of the material, respectively.

[1] P. A. Wolff, Phys. Rev. **95**, 1415 (1954).
[2] W. Shockley, in *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovakian Academy of Sciences, Prague, 1961), p. 81.
[3] G. A. Baraff, Phys. Rev. **133**, A26 (1964).
[4] T. N. Morgan, Bull. Am. Phys. Soc. **2**, 265 (1959).
[5] E. M. Conwell, Phys. Chem. Solids **8**, 234 (1959).
[6] This was first pointed out by F. Seitz, Phys. Rev. **76**, 1376 (1949). Also, W. Shockley, Bell System Tech. J. **30**, 990 (1951), notes that for very high-energy electrons the loss to acoustic modes again "makes an appreciable contribution."
[7] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

The matrix element (2) is obtained by using as the perturbing potential the so-called deformation potential, i.e., the shift of the band edge under the strain due to the acoustic wave. For this to be a good approximation, it was shown in the derivation by Bardeen and Shockley,[8] one must be able to neglect a number of terms of the order of the carrier energy times the strain. This neglect is clearly not as well justified for hot electrons as it is for thermal electrons. We shall show, however, that it should still lead to acceptable results for electrons with, say, $\frac{3}{4}$ V energy in Ge.

One point in the derivation at which such terms are neglected is in the assumption that the strain-produced shift in energy $\delta\mathcal{E}$ of an electron with wave vector $\mathbf{k}$ (relative to the band edge) is the same as the shift of the band edge. To get an estimate of the error involved in this assumption, we shall limit ourselves to terms in k^2, neglecting terms in higher orders of k, and also deal with an isotropic effective mass. The assumption of the same shift for higher levels as for the band edge is then equivalent to the assumption that the effective mass does not change with strain. It is reasonable, however, to expect a change δm in effective mass that is linear in the strain, since only small strains are under consideration. The shift in energy under strain of a state with wave vector $\mathbf{k}+\mathbf{k}^{(i)}$, $\mathbf{k}^{(i)}$ being the wave vector of the band edge, may then be written, to terms linear in strain,

$$\delta\mathcal{E}(\mathbf{k}+\mathbf{k}^{(i)})=\delta\mathcal{E}(\mathbf{k}^{(i)})-\mathcal{E}\delta m/m.$$

The quantity $\mathcal{E}\delta m/m$ has been estimated by Herring[9] for the simple many-valley model to be of the order of $(k/k^{(i)})^2\delta\mathcal{E}(k^{(i)})$. For electrons of 0.75 eV in Ge, the quantity $(k/k^{(i)})^2$ ranges from 0.02 to 0.3 as the mass goes from the observed value of m_t, $0.08m_0$, to that of m_l, $1.6m_0$. Thus it should still be tolerable for 0.75-eV electrons to neglect the additional shift due to change in mass. It may be noted that the quantity $\mathcal{E}\delta m/m$ could be of the order of $\delta\mathcal{E}(\mathbf{k}^{(i)})$ even for thermal electrons in the case of degenerate bands, or of a material in which the effective mass depends strongly on the energy gap.[10]

At other points in the demonstration[8] of the validity of replacing the perturbing potential by the deformation potential there appear terms involving the gradient of the strain. This is proportional to $\mathbf{q}$, which is $\sim\mathbf{k}$, and, therefore, also larger for hot electrons. Bardeen and Shockley have shown that these terms are of the order of $\mathcal{E}\times$strain.[8] Such terms appear in sums that include terms of the order of the deformation potential times strain. Since the deformation potentials in n-Ge are about 10 V or more, one may still neglect the terms in $\mathcal{E}$ for $\frac{3}{4}$-V electrons without serious error. We conclude that the matrix element (2) may still be used with reasonable accuracy for electrons of such energy. In the treatment that follows we also neglect dispersion, and the effect on the energy of terms quadratic in $\mathbf{k}$. This undoubtedly diminishes somewhat the accuracy of $d\mathcal{E}/dt$ for the higher energies, but again the effects are still not large enough to affect the results seriously.

To evaluate (1) we convert $\sum_q$ to an integration in the transformed crystal momentum space in which the constant-energy ellipsoids are spheres. This requires the substitution:

$$q_i=q_i^*(m_i/m_0)^{1/2},\quad \text{for}\quad i=x,y,z, \tag{3}$$

and a similar substitution for the components of $\mathbf{k}$. With (2) and (3) we may write

$$\frac{d\mathcal{E}}{dt}(\mathbf{k})=\frac{m_t m_l^{1/2}\hbar}{8\pi^2\rho m_0^{3/2}}\times\sum_\alpha\int_{q^*}\int_{\theta^*}\int_{\varphi^*}\Xi_\alpha^2 q^2[N_{q\alpha}\delta(\mathcal{E}_{\mathbf{k}+\mathbf{q}}-\mathcal{E}_{\mathbf{k}}-\hbar u_\alpha q)-(N_{q\alpha}+1)\delta(\mathcal{E}_{\mathbf{k}-\mathbf{q}}-\mathcal{E}_{\mathbf{k}}+\hbar u_\alpha q)]đq^*. \tag{4}$$

As in the case of the simple model,[5] emission and absorption due to $N_{q\alpha}$ approximately cancel each other, leaving a small net absorption rate that is significant only for electrons at or near thermal energy. Since our present interest is in high-energy electrons, we shall drop the terms in (4) in $N_{q\alpha}$. When $\mathbf{q}$ is expressed in terms of $\mathbf{q}^*$, (4) becomes

$$\frac{d\mathcal{E}}{dt}(\mathbf{k})=-\frac{m_t m_l^{3/2}\hbar}{8\pi^2\rho m_0^{5/2}}\sum_\alpha\int d\varphi^*\int d\theta^*\sin\theta^*\Xi_\alpha^2 C(\theta^*)\times\int đq^* q^{*4}\delta(\mathcal{E}_{\mathbf{k}-\mathbf{q}}-\mathcal{E}_{\mathbf{k}}+\hbar u_\alpha q), \tag{5}$$

where $C(\theta^*)=\cos^2\theta^*+(m_t/m_l)\sin^2\theta^*$. Because of the dependence of $C(\theta^*)$ and Ξ_α^2 on θ^*, it is necessary to change the usual order of integration[11] and integrate (5) over q^* first. Neglecting terms of the order of $m_0u_\alpha/\hbar k^*$, as is usual, we find

$$\int dq^* q^{*4}\delta(\mathcal{E}_{\mathbf{k}-\mathbf{q}}-\mathcal{E}_{\mathbf{k}}+\hbar u_\alpha q)=\frac{2^4 m_0}{\hbar^2}k^{*3}[\cos\theta^*\cos\theta_k^*+\sin\theta^*\sin\theta_k^*\cos(\varphi^*-\varphi_k^*)]^3, \tag{6}$$

where θ_k^* and φ_k^* denote the polar and azimuthal angles of $\mathbf{k}^*$. As a result of (6), $(d\mathcal{E}/dt)(\mathbf{k})$ varies with the direction of $\mathbf{k}$ or $\mathbf{k}^*$. To obtain the average of $d\mathcal{E}/dt$ over $\mathbf{k}^*$, one may carry out the integration for $\mathbf{k}^*\|z$, and for $\mathbf{k}^*\perp z$, and average the results with appropriate weighting. In order that q^* be positive where the argument of the δ function vanishes, it is necessary to restrict the ranges of integration over θ^* and φ^*. Finally, we find for

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[9] C. Herring, Bell System Tech. J. **34**, 237 (1955).
[10] R. W. Keyes, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1960), Vol. 11, p. 149.
[11] See, for example, E. M. Conwell, Phys. Rev. **135**, A814 (1964).

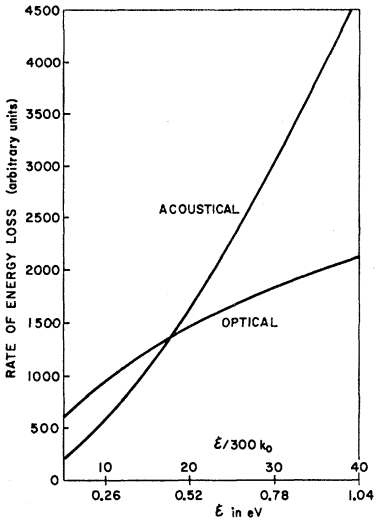

FIG. 1. Rates of energy loss of electrons in Ge to intravalley acoustic and optical modes at a lattice temperature of 300°.

the average rate of energy loss due to intravalley acoustic scattering of electrons with energies several times thermal or greater:

$$(d\mathcal{E}/dt)_{\rm ac}=-(2^{3/2}m_t^2m_l^{1/2}\Xi_0^2/\pi\hbar^4\rho)\mathcal{E}^{3/2}, \qquad (7)$$

where $\Xi_0^2=\Xi_d^2[\frac{2}{3}+\frac{1}{3}(m_l/m_t)((\Xi_u/\Xi_d)+1)^2]$. If the usual substitutions are made to take the many-valley model into the simple model—i.e., $m_l=m_t=m$, $\Xi_u=0$ and $\Xi_d=E_1$, the deformation potential for the simple model—it is seen that (7) goes over to the expression obtained earlier for that case.[12] It is also seen that if the electrons in the valley under consideration are assumed to have a Maxwell-Boltzmann energy distribution at a temperature T_e, the average over the distribution of $d\mathcal{E}/dt$ given by (7) agrees with the result given earlier[13] for the average rate of energy loss.

The rate of energy loss to optical modes may also be calculated from the expression (1). Since the matrix element and the phonon energy for that case are independent of θ, the calculation is very much the same as for the simple model of the band structure,[5] the only difference being the replacement of the simple model effective mass by the geometric mean mass. For $\mathcal{E}>\hbar\omega_0$, the optical phonon energy, the result may be stated

$$\left(\frac{d\mathcal{E}}{dt}\right)_{\rm op}=-\frac{1}{\sqrt{2}}\frac{m_tm_l^{1/2}D^2}{\pi\hbar^2\rho}\frac{e^{x_0}}{e^{x_0}-1}\times[(\mathcal{E}-\hbar\omega_0)^{1/2}-e^{-x_0}(\mathcal{E}+\hbar\omega_0)^{1/2}], \qquad (8)$$

where $x_0=\hbar\omega_0/k_0T$ and D is the optical deformation potential defined as the shift of the band edge per unit displacement of the two sublattices relative to each other. It may be noted that this expression will also be somewhat less accurate at the higher electron energies because additional terms in the matrix element, of order k,[14] will begin to be significant, as well as dispersion and quadratic terms in $\mathcal{E}(\mathbf{k})$.

Values of D for electrons in germanium have been obtained from the interpretation of infrared absorption data,[15] and from the saturation drift velocity of hot electrons.[16] The two values are quite similar. Since in both cases the D value obtained depends on the values of Ξ_d and Ξ_u assumed, it is advisable to choose the complete set from one source or the other. We have chosen the set of de Veer and Meyer[15]: $D=4\times10^8$ eV/cm, $\Xi_d=-9.07$ eV, and $\Xi_u=19.3$ eV. In the other set D is larger, but the Ξ_d and Ξ_u values lead to a somewhat larger value of Ξ_0, so the ratio of optical to acoustic mode scattering is increased only a little. For the other quantities that appear in (7) and (8) we have used the usual values: $\hbar\omega_0/k=400°$, $m_t=0.082m_0$, $m_l/m_t=20$. The resulting rates of energy loss at room temperature are plotted in Fig. 1. It is seen that the rate of loss to acoustic modes begins to exceed that to optical modes at electron energies less than the gap energy. This should also be true at 78° since the loss rates are practically independent of temperature.

The value of D given above contains also the effect of the small amount of equivalent intervalley scattering that one expects in Ge.[15] It does not, however, contain any contribution of nonequivalent intervalley scattering. It has been suggested[17] that scattering from the [111] valleys to the [100] valleys that lie 0.2 eV higher may be important for hot electrons in Ge. De Veer and Meyer find that the deformation potential constant for this process is comparable in magnitude to D, and that the average mass in the [100] valleys is about 35% of that for the [100] minima in silicon. This suggests that the energy loss due to this process may also have to be taken into account for high-energy electrons.

For holes in germanium, because of the stronger coupling to the optical modes, losses to optical modes will outweigh those to acoustic modes to higher energies than has been found to be the case for electrons.

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